THE TEMPERATURE FIELD OF A PLATE IN A SOLIDIFIED GAS

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We examine the problem of finding the temperature field of a plate heated by a constant flow of heat from one end and releasing heat to the solidified gas from the remaining sides through a gas interlayer.

In certain cases we are confronted with the problem of finding the temperature field of a plate heated from one end and giving off heat to a solidified gas from the remaining sides. The dimensions and shape of the plate are shown in Fig. 1a.

A constant pressure – below the pressure of the triple point – is usually maintained over the surface of a solidified gas. Then, because of the limited thermal conductivity of solidified gases, the heat from the plate cannot be removed by conduction and an interlayer is formed about the plate as a consequence of the sublimation of the solid phase. The growth in the thickness of this interlayer impairs the transfer of heat between the plate and the medium and results in the gradual heating of the plate.

To simplify the problem we will make the following assumptions:

1. The heat load on the plate will be assumed to be a small and constant quantity, i.e., $q_x(x, \tau)|_{x=0} = \text{const} = q_0$ (bearing in mind that the error introduced by these assumptions is no greater than 5%, we took q_0 to be less than 0.3 W/cm^2).

2. On the strength of condition 1 we will assume that the plate temperature T varies over a small interval.

3. The thermal conductivity λ and the specific heat capacity c is assumed constant for the entire plate.

4. Because of the limited plate thickness δ we neglect the transfer of heat between the plate and the medium in the direction of the y-axis.

5. Considering condition 4, the limited thickness and high thermal conductivity of the plate, we assume that the temperature does not change through the width and thickness of the plate and is a function exclusively of the coordinate x and of the time τ; $T = T(x, \tau)$.

6. For the plate heat transfer in the direction of the z-axis we will substitute internal negative heat sources of the same intensity, i.e., $q_z(x, \tau)$.

Derivation of the differential heat-conduction equation for the plate is accomplished in analogy with [1]. Let us examine the heat balance of the volume element shown in Fig. 1b:

$$-\frac{\partial q_x}{\partial x}dx\,\delta b - 2q_z dxb = c\rho\frac{\partial T}{\partial \tau}dxb\delta. \tag{1}$$

Using the heat-conduction equation $q_x = -\lambda(\partial T/\partial x)$, we find

$$c\rho\frac{\partial T(x,\ \tau)}{\partial \tau} = \lambda\frac{\partial^2 T(x,\ \tau)}{\partial x^2} - \frac{2}{\delta}q_z(x,\ \tau). \tag{2}$$

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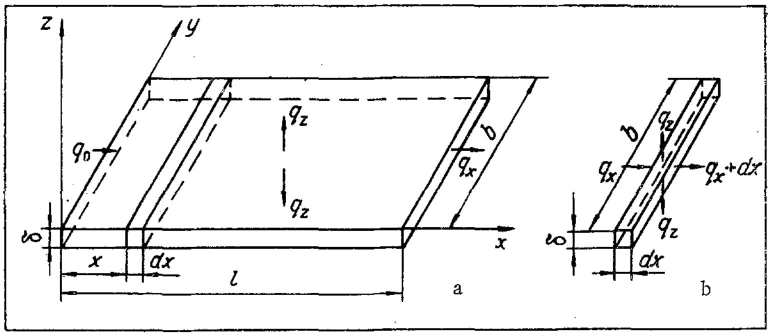

Fig. 1. Selected diagram and conditions of the problem.

To find q_z we examine the transfer of heat at the side surface lb of the plate at some arbitrary instant of time. As demonstrated in Fig. 2, a gas clearance with a thickness of $h(x, \tau)$ is formed about the plate.

7. We will assume that the transfer of heat through this interlayer from the plate to the solidified gas can be described by the heat-conduction equation, using the effective coefficient of thermal conductivity for the gas, i.e., $\varkappa_{eff} = k\varkappa$, considering the other means of heat and mass transfer (convection and heat transfer by means of the sublimating vapors) [2-5]. We can then write

$$q_z = \frac{T(x, \tau) - T_0}{h(x, \tau)} \varkappa_{eff}, \tag{3}$$

where T_0 is the solid-phase temperature which is assumed to be constant, since the pressure p = const.

8. The effective thermal conductivity $\varkappa_{eff}$ of the gas will be assumed to be constant in each specific test, in the light of assumption 2.

9. The thickness h of the gas clearance is found on the assumption that the flow of heat from the surface lb in the direction of the z-axis goes entirely to the sublimation of the solid phase, i.e., the heat capacity of the gas is also neglected in the light of assumption 2.

Equating the heat $\int_0^\tau qz(x, \tau)d\tau$ evolved from a unit surface of the plate during the time τ to the heat of sublimation of the solid gas beneath this unit surface, we find the relationship

$$\int_0^\tau q_z(x, \tau)\, d\tau = \rho_0 r_0 h(x, \tau). \tag{4}$$

Using (3), we write (2) in the form

$$c\rho \frac{\partial T(x, \tau)}{\partial \tau} = \lambda \frac{\partial^2 T(x, \tau)}{\partial x^2} - \frac{2}{\delta} \frac{T(x, \tau) - T_0}{h(x, \tau)} \varkappa_{eff}. \tag{5}$$

Let us find the boundary-value problems for (5). We assume that the plate temperature at the initial instant of time is equal to the solid-phase temperature, i.e.,

$$T(x, 0) = T_0.$$

To find the boundary condition for x = 0 we use the condition

$$q_x(0, \tau) = q_0 = -\lambda \frac{\partial T}{\partial x}(0, \tau).$$

Neglecting the heat transfer at the end of the plate when $x = l$, because of the smallness of its area δb as compared to the side surface $2bl$, we find

$$q_x(l, \tau) = -\lambda \frac{\partial T}{\partial x}(l, \tau) = 0.$$

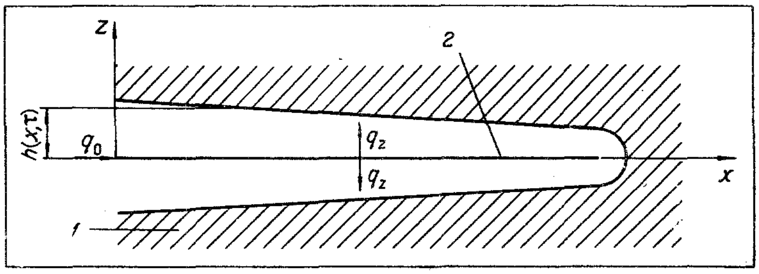

Fig. 2. Proposed shape of the gas clearance about the plate: 1) solidified gas; 2) plate.

The problem thus reduces to the solution of (5) for the following boundary conditions:

$$T(x,\ 0) = T_0,\quad T(x,\ \tau) - T_0 \geqslant 0,\quad \tau \geqslant 0; \tag{6}$$

$$\frac{\partial T}{\partial x}(0,\ \tau) = -\frac{q_0}{\lambda}; \tag{7}$$

$$\frac{\partial T}{\partial x}(l,\ \tau) = 0. \tag{8}$$

An M-20 digital computer was used to solve (5) on the basis of an implicit scheme propsed by Krank and Nicholson; the error in the approximation of the difference scheme in this case is of the order of

$$e = O\,[(\Delta\tau)^2] + O\,[(\Delta x)^2] = O\,(0.001^2) + O\,(0.025^2).$$

It is convenient to present $h(x,\ \tau)$ in the form

$$h(x,\ \tau) = h(x,\ \tau - \Delta\tau) + \frac{T(x,\ \tau) - T_0}{\rho_0 r_0 h(x,\ \tau - \Delta\tau)}\varkappa_{\text{eff}}\Delta\tau. \tag{9}$$

The results of the solution showed that the plate very rapidly (in less than 0.5 h) enters a thermal regime close to the regular, during which the temperature at each point on the plate rises linearly with time, while the temperature distribution $T(x,\ \tau) - T(l,\ \tau)$ and the temperature difference across the length of the plate are independent of time. After entry into the regular regime, we find that q_z virtually ceases to depend on the coordinate x. This can be explained by the constancy of q_0 and by the fact that – because of the smallness of the ratio q_0/λ – the temperature difference across the length of the plate is small in comparison with the difference between the plate and the solid gas. The indicated constancy of qz is in good agreement with the constancy of the temperature difference across the length of the plate.

We will give an approximate analytical solution of the problem, using the constancy of q_z over the length of the plate.

On the basis of condition 2 we will neglect the heat spent on altering the heat capacity of the plate as small in comparison with the heat expended on sublimation. The heat-balance equation for the entire plate then yields

$$q_z = \frac{q_0}{2}\frac{\delta}{l}, \tag{10}$$

while (4) assumes the form

$$q_z = \frac{\rho_0 r_0 h(\tau)}{\tau}. \tag{11}$$

Equation (2) under these assumptions can be simplified

$$\lambda\frac{d^2T(x,\ \tau)}{dx^2} = \frac{q_0}{l}. \tag{12}$$

After double integration of this equation with the use of (7), we have

$$T = \frac{q_0}{\lambda}\frac{x^2}{2l} - \frac{q_0}{\lambda}x + c_1.$$

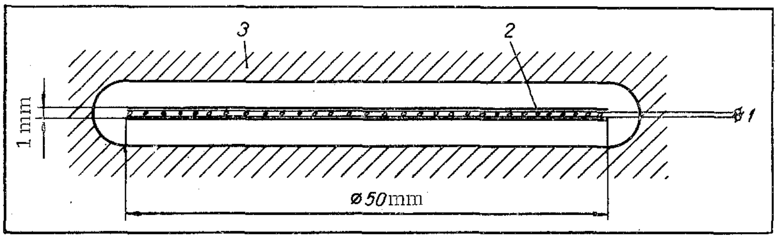

Fig. 3. Arrangement of the plate and the shape of the gas clearance about it: 1) heater; 2) plate; 3) solidified gas.

To determine the integration constant c_1, let us examine the mean-integral temperature over the length of the plate; this is equal to

$$T_m = \frac{1}{l}\int_0^l T(x, \tau)\,dx = c_1 - \frac{q_0}{\lambda}\frac{l}{3}.$$

On the other hand, the mean-integral temperature can be found from (3), (10), and (11):

$$T_m = \frac{q_0^2}{4}\frac{\delta^2}{l^2}\frac{1}{\rho_0 r_0 \varkappa_{eff}}\tau + T_0. \tag{13}$$

From the last three equations, the expression for the temperature field of the plate assumes the form

$$T(x, \tau) = T_0 + \frac{q_0}{\lambda}\frac{x^2}{2l} - \frac{q_0}{\lambda}x + \frac{q_0}{\lambda}\frac{l}{3} + \frac{q_0^2}{4}\frac{\delta^2}{l^2}\frac{1}{\rho_0 r_0 \varkappa_{eff}}\tau. \tag{14}$$

It follows from (14) that the temperature difference across the length of the plate is constant in time, i.e.,

$$T(0, \tau) - T(l, \tau) = \frac{q_0}{\lambda}\frac{l}{2}. \tag{15}$$

Since (14) and (15) have been derived on the assumption that q_z = const, they will be valid on elapse of a certain period of time from the onset of heating, at which time we have a regime that is close to the regular.

The approximate solution (14) differs from the exact solution given by the computer by no more than 3-5%, but it substantially simplifies the calculations of the plate temperatures.

We performed a number of experiments to determine the effective coefficient of heat transfer between the plate and the solidified gas.

Solid nitrogen is used as the cooling agent; the test plate is frozen into the solid nitrogen, as shown in Fig. 3. The plate is fashioned of two parts with a thickness of 0.1 mm, with a double-helical heater attached in such a manner that it is not in electrical contact with either part of the plate. Differential copper–constantan thermocouples were used to measure the temperature difference between the plate and the solid cooling agent with a specified heater power which was determined by measuring the current and voltage in the circuit. The stability of the heat supply was ±1%. The temperature difference was measured with an accuracy of ±0.05°K.

To reduce spurious influxes of heat to the test plate, all of the leads were kept to a diameter of 0.1 mm and, in addition, at some distance from the plate (of the order of 150 mm) the leads were interrupted with a temperature control set for a temperature close to that of the test plate. An experiment with a zero load on the plate showed that the plate temperature within the limits of measurement accuracy remains constant and equal to T_0 for 15-20 h, i.e., there is virtually no spurious influx of heat.

Because of the smallness of the end surface in comparison with the side surfaces, we can neglect the transfer of heat at the end of the plate. Bearing this in mind, as well as the excellent thermal conductivity of the plate material, in addition to the fact that the heater was wound uniformly over the entire surface, we can assume that the plate exhibited an identical temperature over the entire surface In this connection, the plate temperature will be determined from (13), which can be presented in the form

$$T(\tau) - T_0 = \frac{1}{4\varkappa_{eff}\,\rho_0 r_0}\left(\frac{Q_0}{F}\right)^2\tau. \tag{16}$$

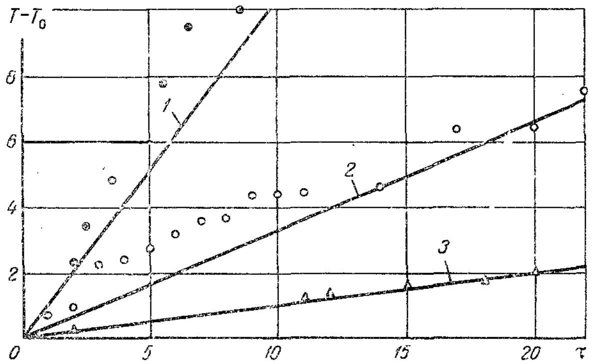

Fig. 4. Time variation in the temperature gradient between the horizontal plate and the solidified gas for various heat loads, $T - T_0$, in °K; τ, in h; Q_0, in W: 1) Q = 0.1; 2) 0.0563; 3) 0.0314; solid lines denote the theoretical relationship from (16) for k = 1.5; the dots denote the experiment.

During the test, at a constant heat load Q_0 on the plate, we measured the temperature difference $T(\tau) - T_0$ over time. The tests were performed at various loads Q_0.

The experiment showed that the plate temperature can be described by (16), but for each specific case we must know the value of the effective coefficient of thermal conductivity $\varkappa_{eff}$, which, because of the complexity of the mechanism of heat transfer to the sublimating medium, is a function of numerous variables. In the general case, this coefficient proves not to be equal to the coefficient of thermal conductivity for the gas and is a function of the pressure of the saturated vapors, the position (vertical or horizontal), and the dimensions of the heat-release surface and the magnitude of the heat load Q_0 on the specimen, which in turn determines the rate of sublimation and the velocity of the forced motion of the gas in the clearance. The experimental derived values for $\varkappa_{eff}$ varied in a wide range from k = 1 to 4. Here the thermal conductivity of the gaseous nitrogen was assumed to be equal to $\varkappa = 0.007$ W/deg·m. On a vertical plate the value of $\varkappa_{eff}$ proved to be greater by a factor of 1.3-2 than on a horizontal plate. Relationship (16) is compared in Fig. 4 with the experiment for a pressure of p = 12-15 mm Hg for the saturated nitrogen vapors.

As we can see from Fig. 4, the greatest divergence from (6) is found for loads of 0.1 and 0.0563 W, which go beyond the limits of the adopted assumptions. At smaller loads (0.0314 and less, Fig. 4), when the perturbing effect of convection flows is weaker and the clearances are smaller, the agreement between the experiment and (16) is better, and the quantity $\varkappa_{eff}$ itself approaches the thermal conductivity of the gas. The experiments show that at smaller loads there is also a reduced effect on the part of plate position. The derived equation (14) must thus better approximate the real distribution of the temperature with a reduction in q_z below 0.001 W/cm^2, with the value of $\varkappa_{eff}$ in this case tending toward the coefficient of thermal conductivity $\varkappa$. At greater loads, the mechanism of heat transfer through the gas clearance to the sublimating medium is substantially complicated and for the determination of $\varkappa_{eff}$ in this case we have to perform additional experiments.

NOTATION

x, y, x	are coordinates;
τ	is the time;
q	is the specific heat flow;
$q_0 = Q_0/\delta b$	is the specific heat flow to the end of the plate;
c, ρ	are, respectively, the specific heat capacity and the specific density of the plate material;
ρ_0, r_0	are, respectively, the specific density and heat of sublimation for the solid gas;
F	is the area of the side surface of the plate;
$\varkappa$	is the coefficient of thermal conductivity for the gas;
k	is the coefficient by means of which we take into consideration the effect of various factors on the transfer of heat through the gas;
l, δ, b	are, respectively, the length, the thickness, and the width of the plate.

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